

---

**System I**

**Computer System and Information  
Representation**

Haoting Shen

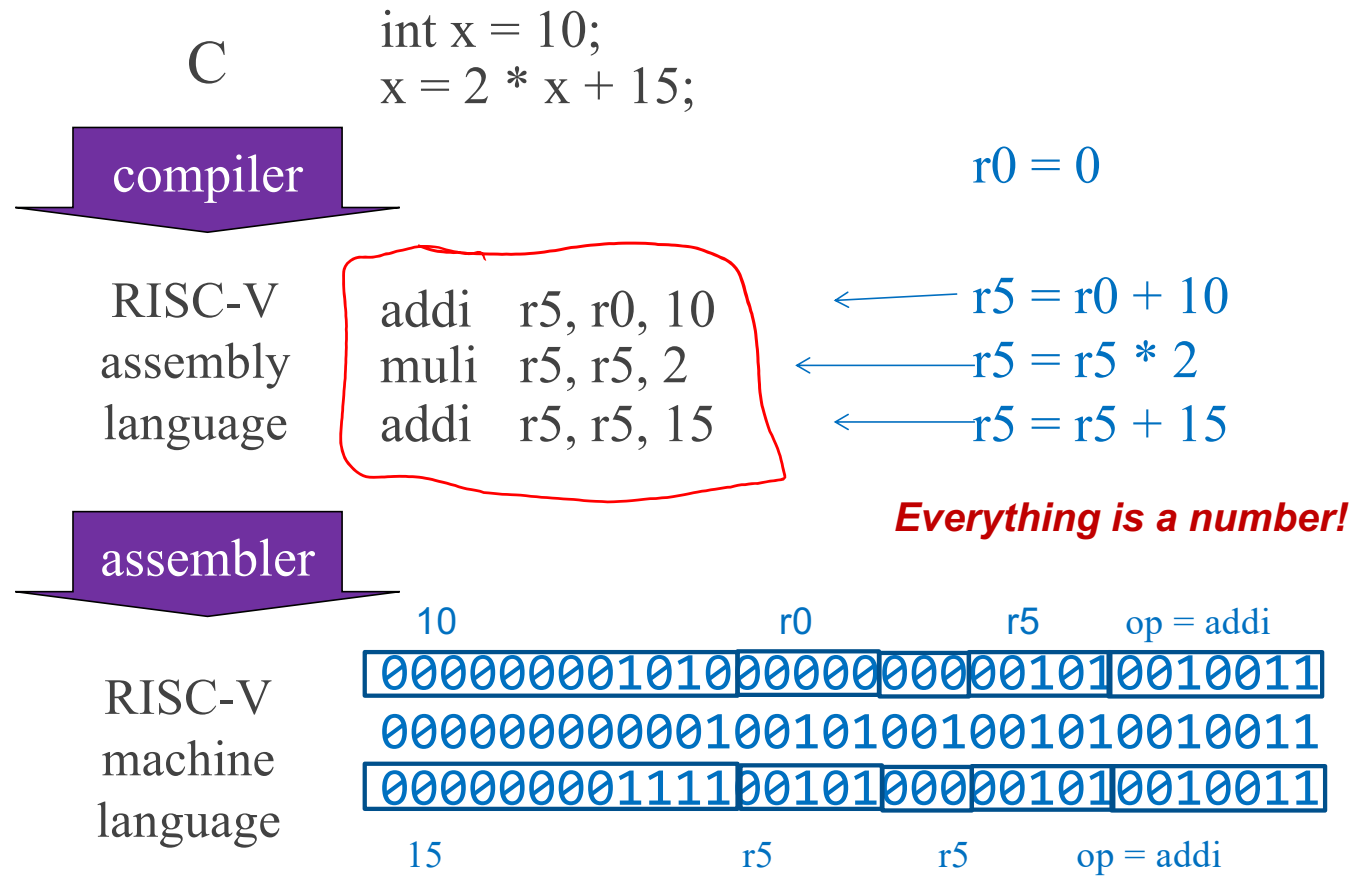
Zhejiang University

# Disclaimer

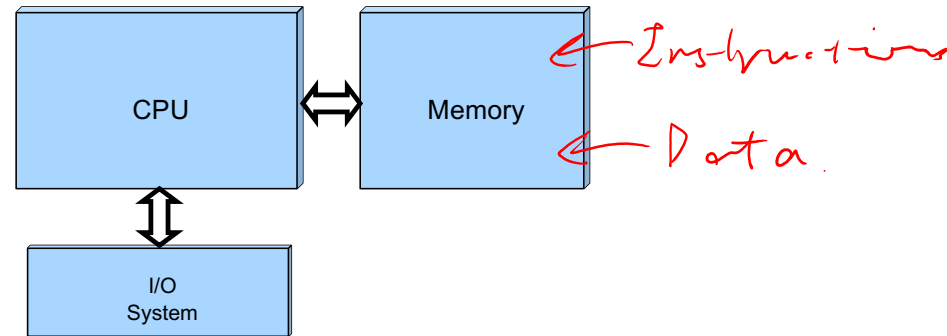
---

- **Many images and resources used in this lecture are collected from the Internet, and they are used only for the educational purpose. The copyright belong to the original owners, respectively.**
  
- **Part of slides credit to**
  - **CSAPP @ CMU**
  - **Prof. Yabo Dong @ ZJU**
  - **Dr. Jun Lu, Peiking Univ.**
  - **Coursera.org**

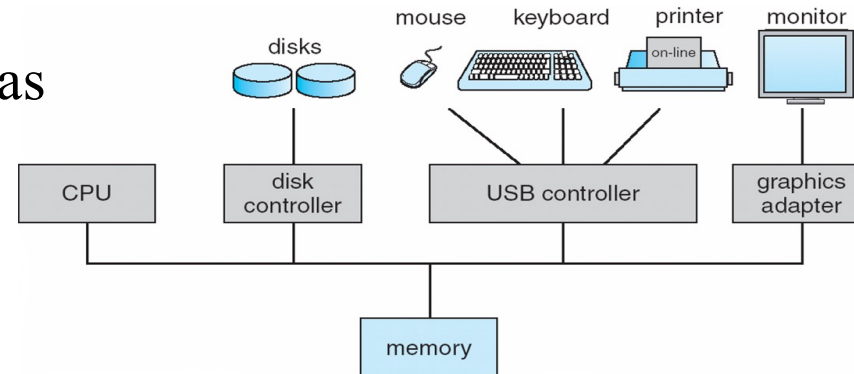
# Source Code -> Executable File



# Von-Neumann Model



- Amazingly, it's still possible to think of the computer this way at a conceptual level (model from ~80 years ago!!!)
- A computer today has some differences



## Conceptual View of Memory

---

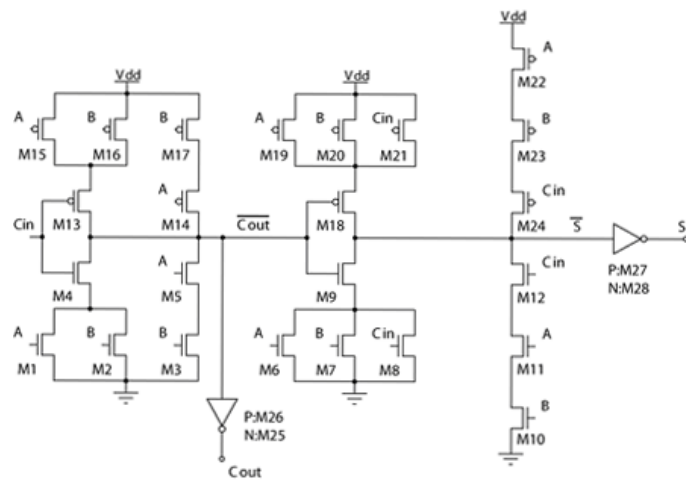
address	content
0000 0000 0000 0000	0110 1110
0000 0000 0000 0001	1111 0100
<b>0000 0000 0000 0010</b>	<b>0000 0000</b>
0000 0000 0000 0011	0000 0000
0000 0000 0000 0100	0101 1110
0000 0000 0000 0101	0000 0000
0000 0000 0000 0110	0000 0000
0000 0000 0000 0111	0100 0000
0000 0000 0000 1000	1111 0101

**At address 0000 0000 0000 0010  
the content is 0000 0000**

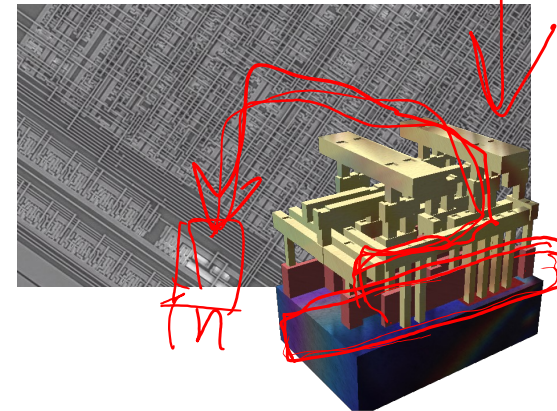
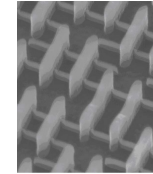
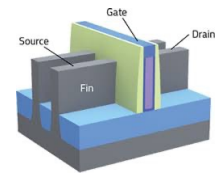
...

...

## Transistor Schematic



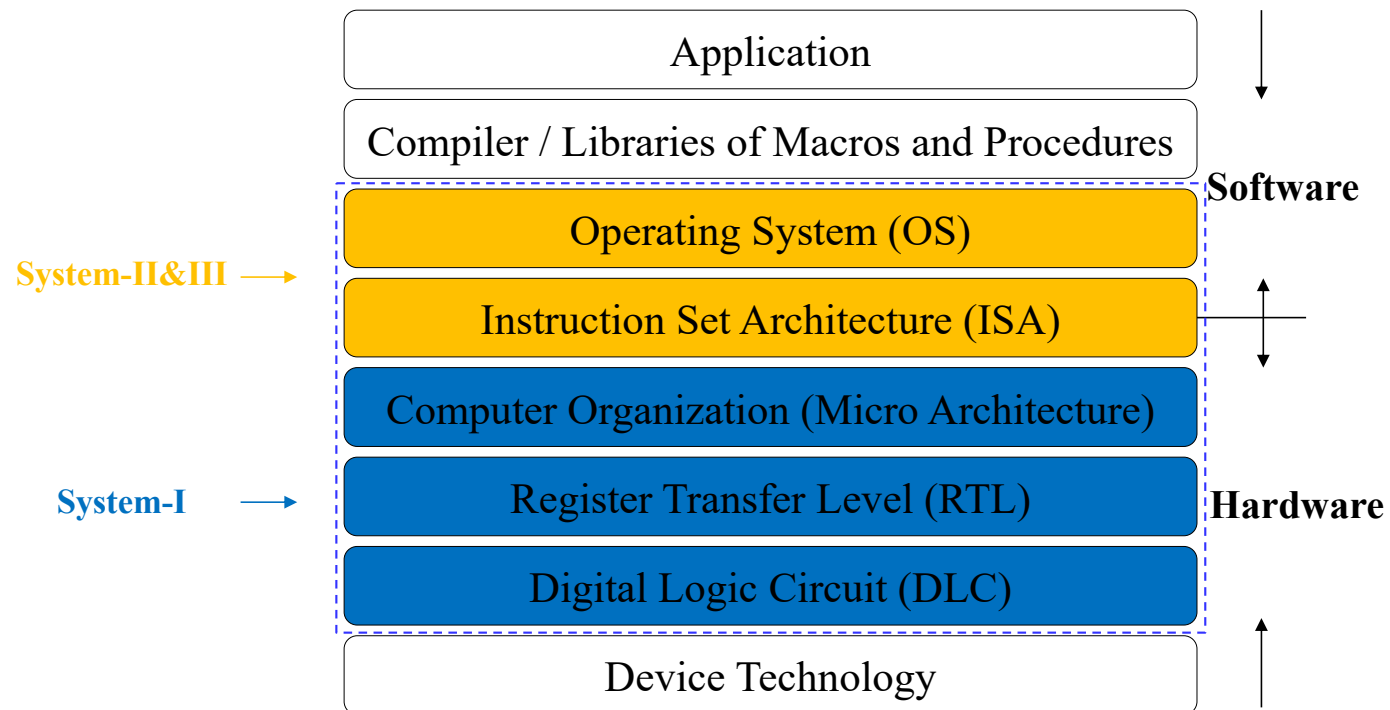
Full adder



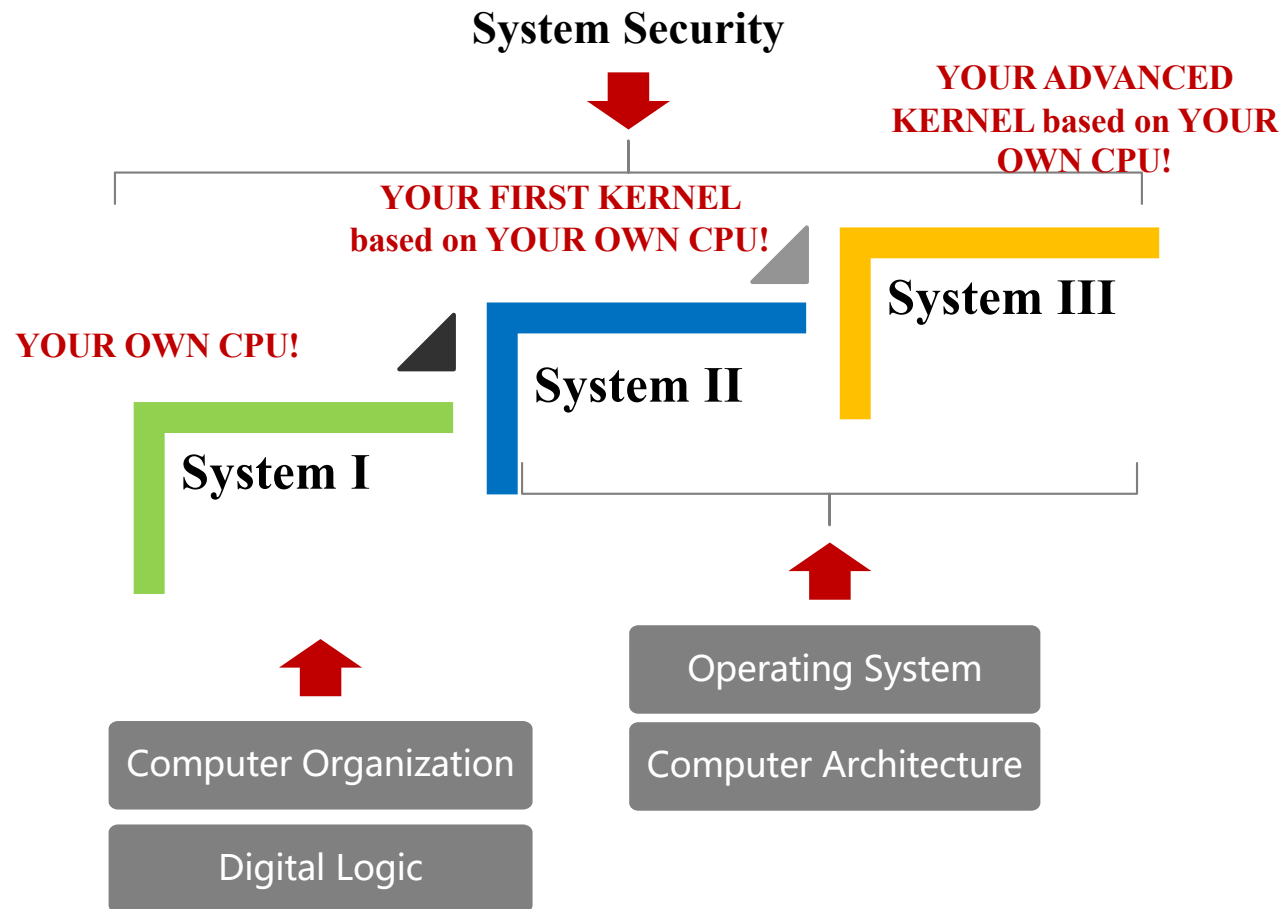
Very Large Scale Integrated (VLSI) Circuit

# Covered in ZJU Computer System Courses

---



# ZJU Computer System Courses

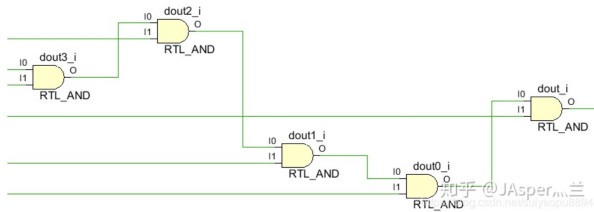


# LUT

```
// 以下是例1
module top(
    input clk,
    input rst,
    input data0,
    input data1,
    input data2,
    input data3,
    input data4,
    input data5,
    output dout
);

    assign dout = data0&data1&data2&data3&data4&data5;

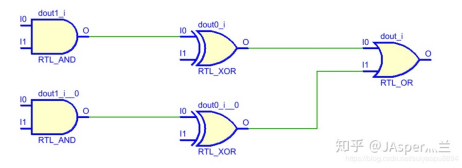
endmodule
```



```
// 以下是例2
module top(
    input clk,
    input rst,
    input data0,
    input data1,
    input data2,
    input data3,
    input data4,
    input data5,
    output dout
);

    assign dout = ((data0&data1)^data2|data3&data4^data5);

endmodule
```



# Overview

---

- **Introduction to computer systems**
- Binary number representation
- Arithmetic operations
- Representation of numeric data
- Representation of non-numeric data
- Data width and storage

# ENIAC

---

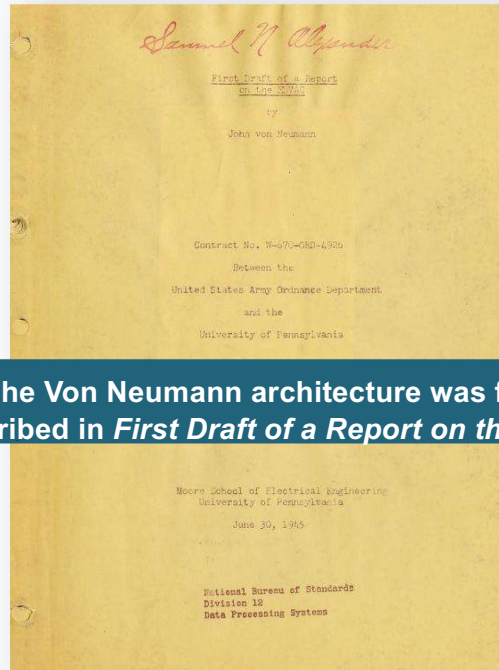
- ENIAC (Electronic Numerical Integrator and Computer)
- EE@Upenn
- Funded by the U.S. Army's Ballistics Research Laboratory to calculate missiles

**Use mechanical switches, manual wire connections, and other settings to implement instructions.**



ENIAC (图片来自维基百科)

# Von Neumann and EDVAC



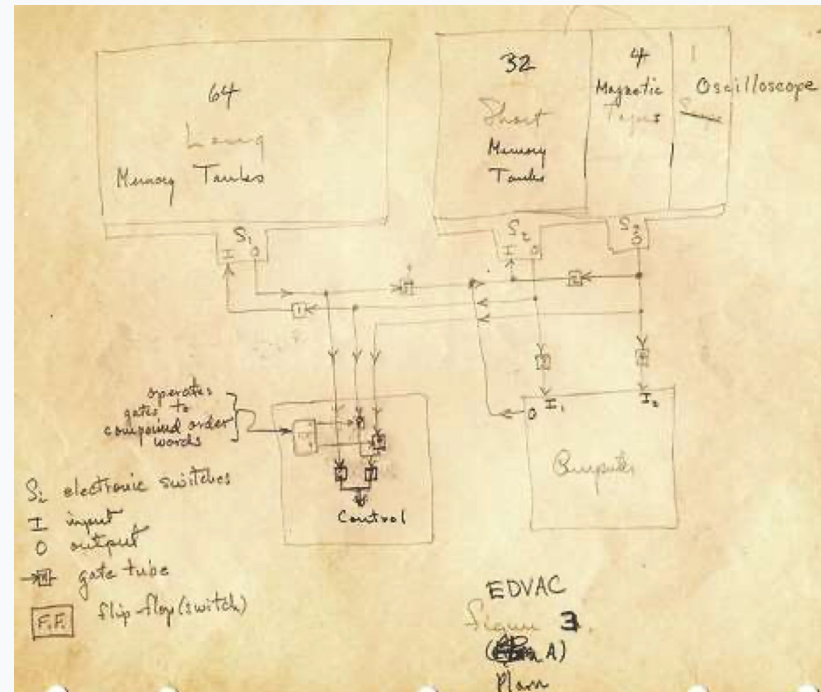
**The Von Neumann architecture was firstly described in *First Draft of a Report on the EDVAC***



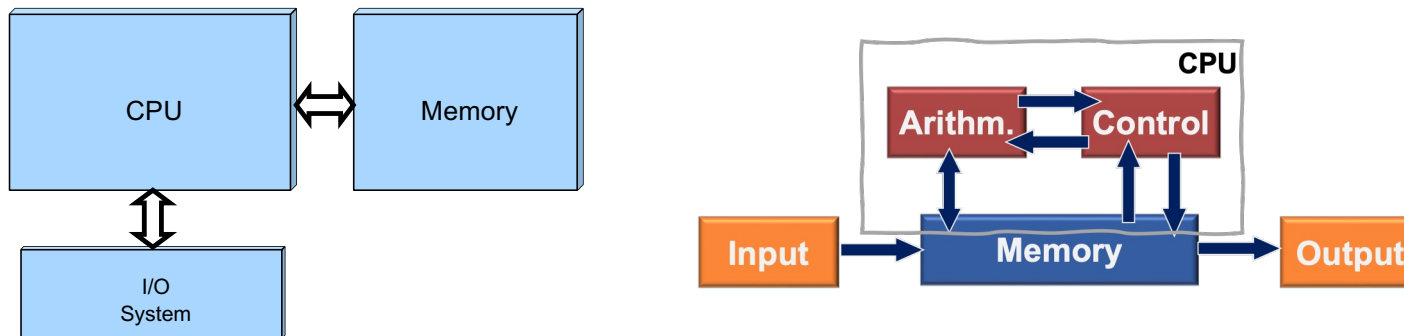
**John Von Neumann  
1903~1957**

# Von Neumann and EDVAC

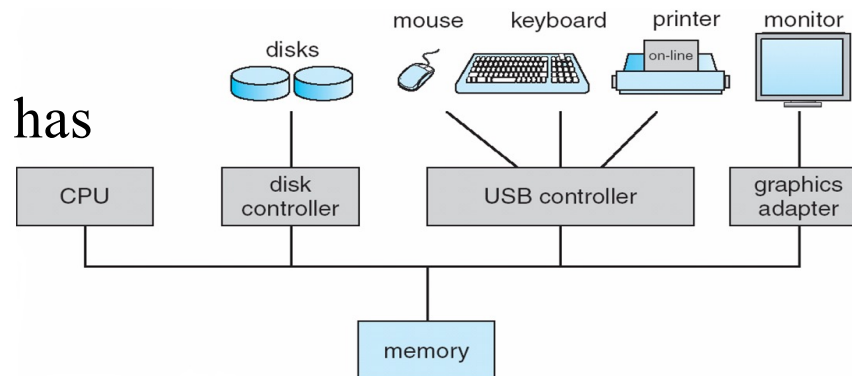
- 101 pages, unfinished manuscript
- Two important conceptions
  - **Stored program**
  - Binary
- 5 parts
  - Central Arithmetical (CA)
  - Central Control (CC)
  - Memory (M)
  - Input (I)
  - Output (O)



# Von-Neumann Model

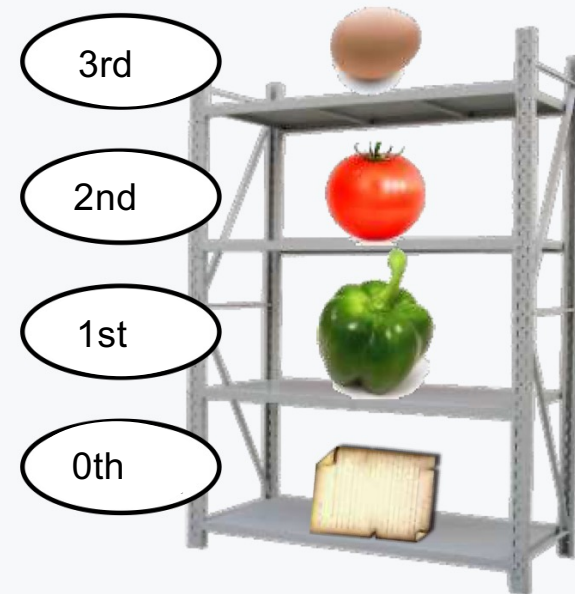


- Amazingly, it's still possible to think of the computer this way at a conceptual level (model from ~80 years ago!!!)
- A computer today has some differences



# Main Memory (also called RAM)

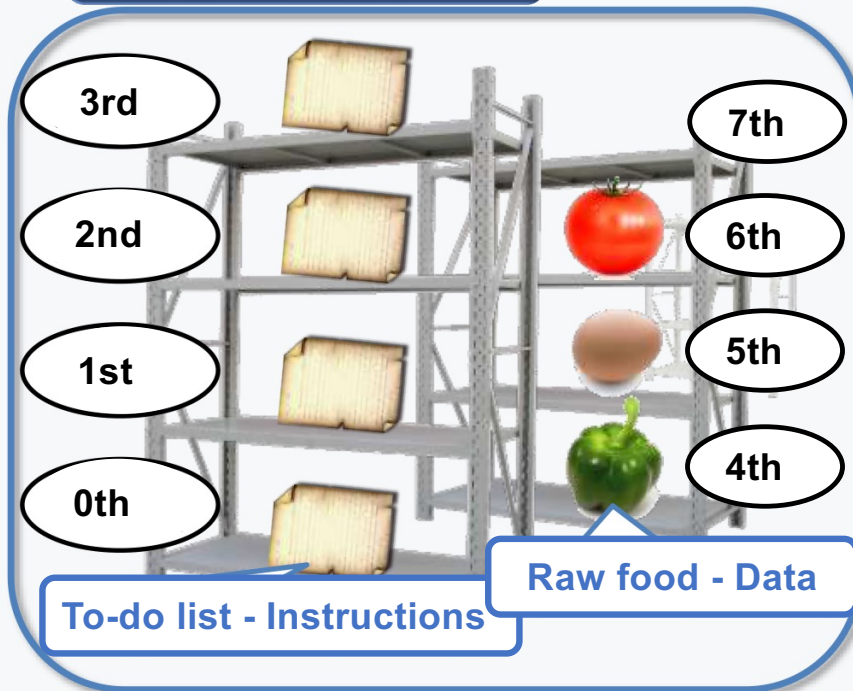
	Address	Content
High address	...	.....
	0011	00001100
	0010	00100010
Low address	0001	00000000
	0000	01101101



# Von Neumann Computer Vs. Restaurant

## Restaurant - Computer

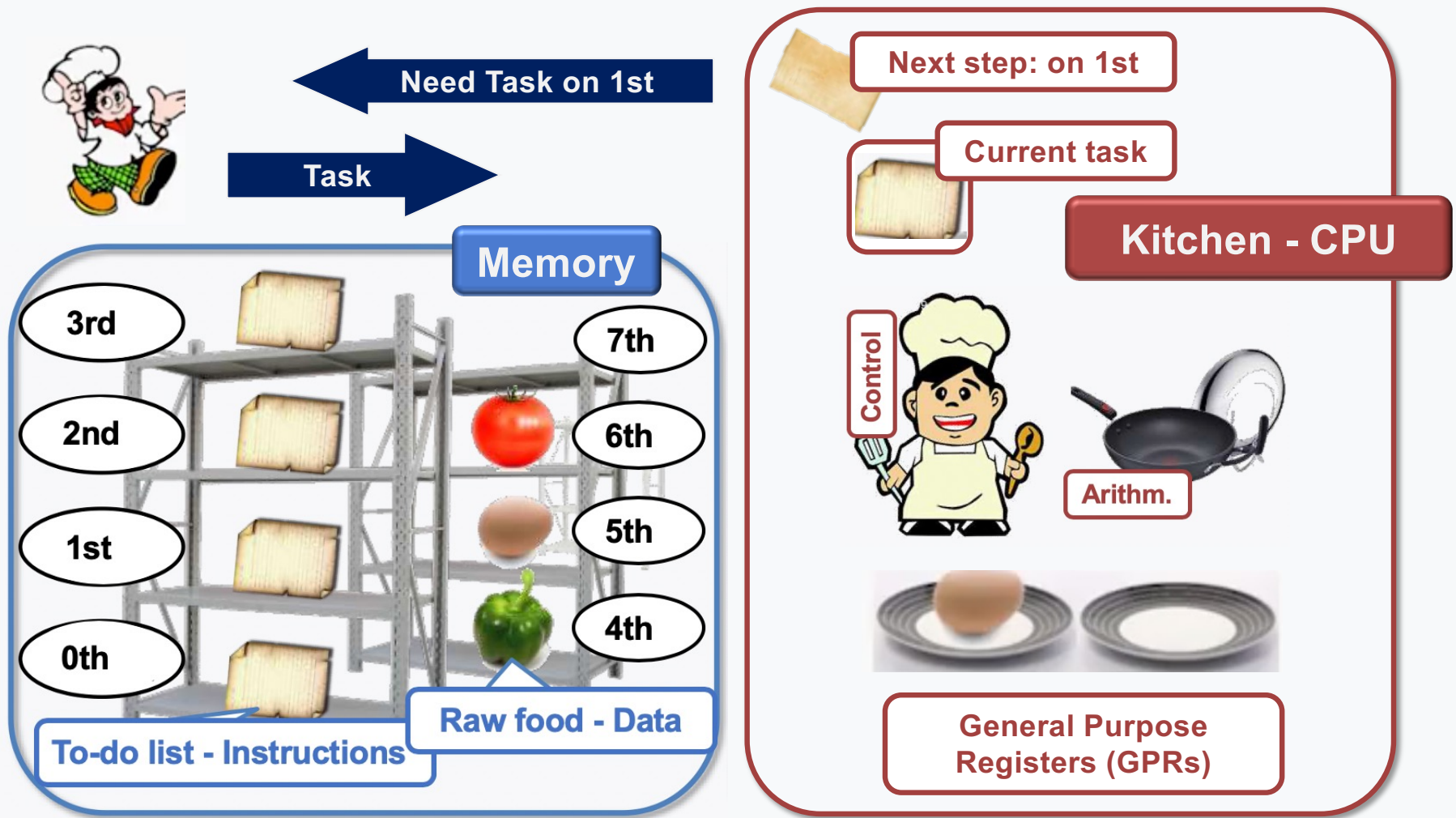
### Storage - Memory



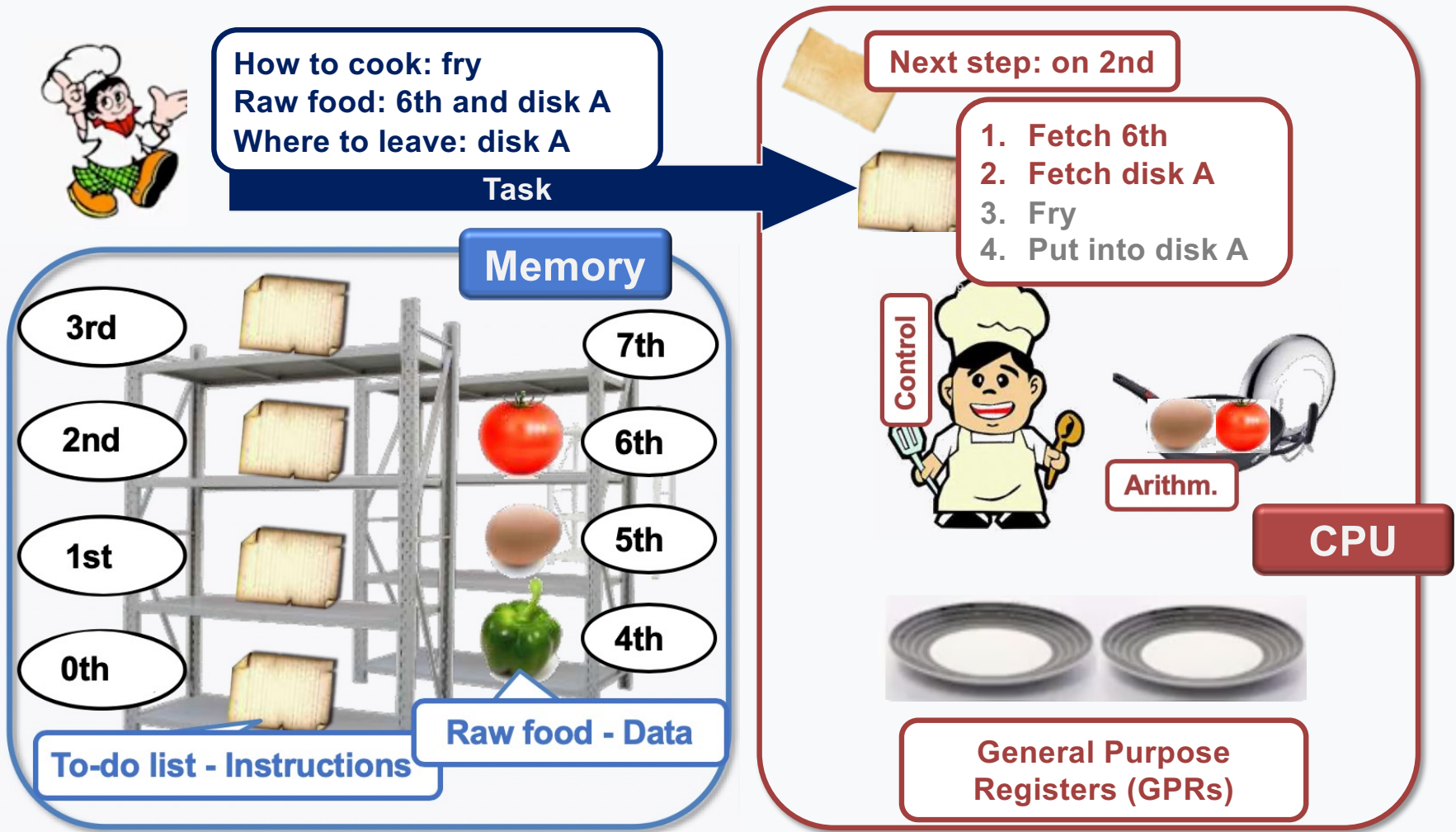
### Kitchen - CPU



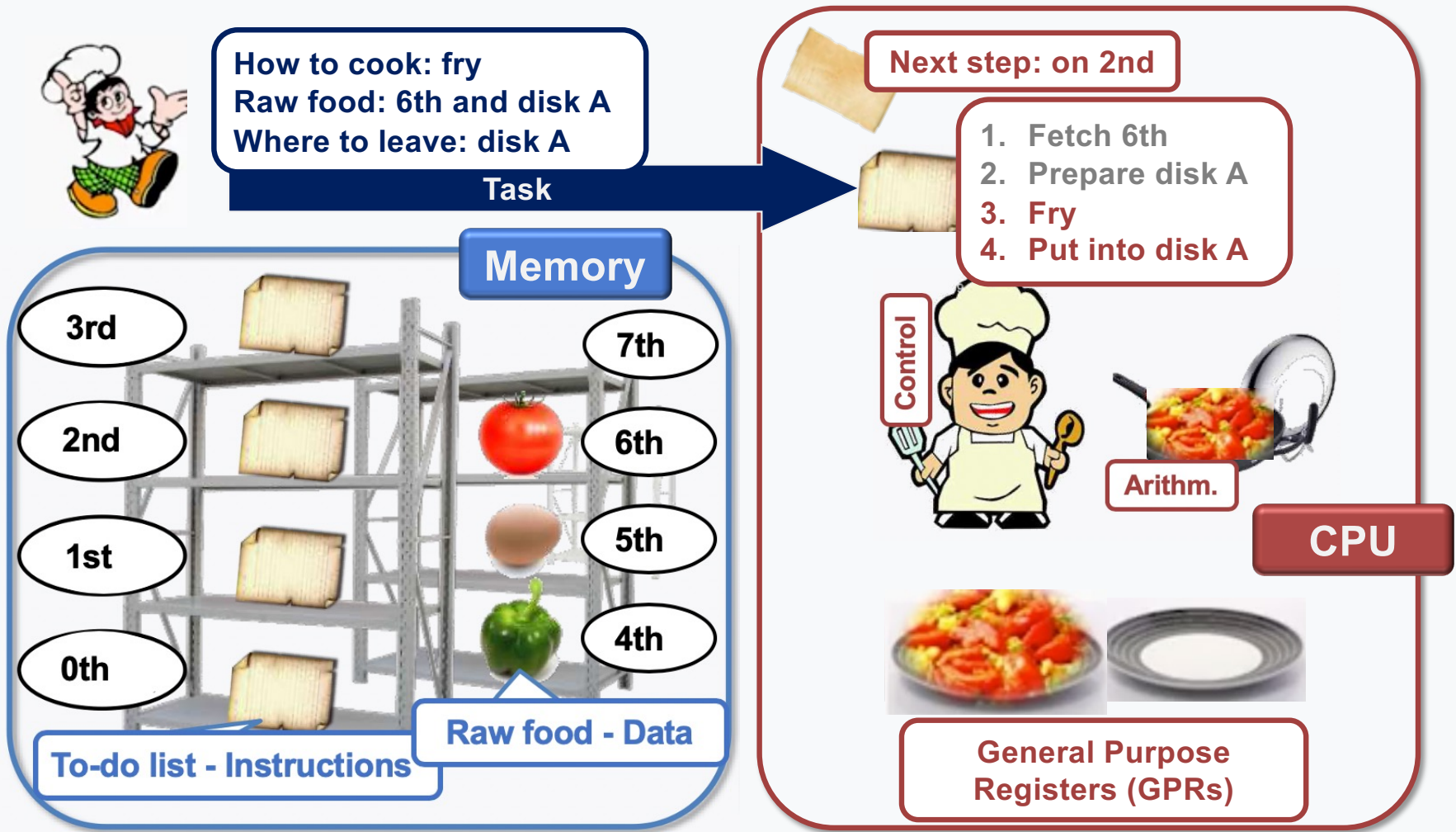
# Von Neumann Computer Vs. Restaurant



# Von Neumann Computer Vs. Restaurant

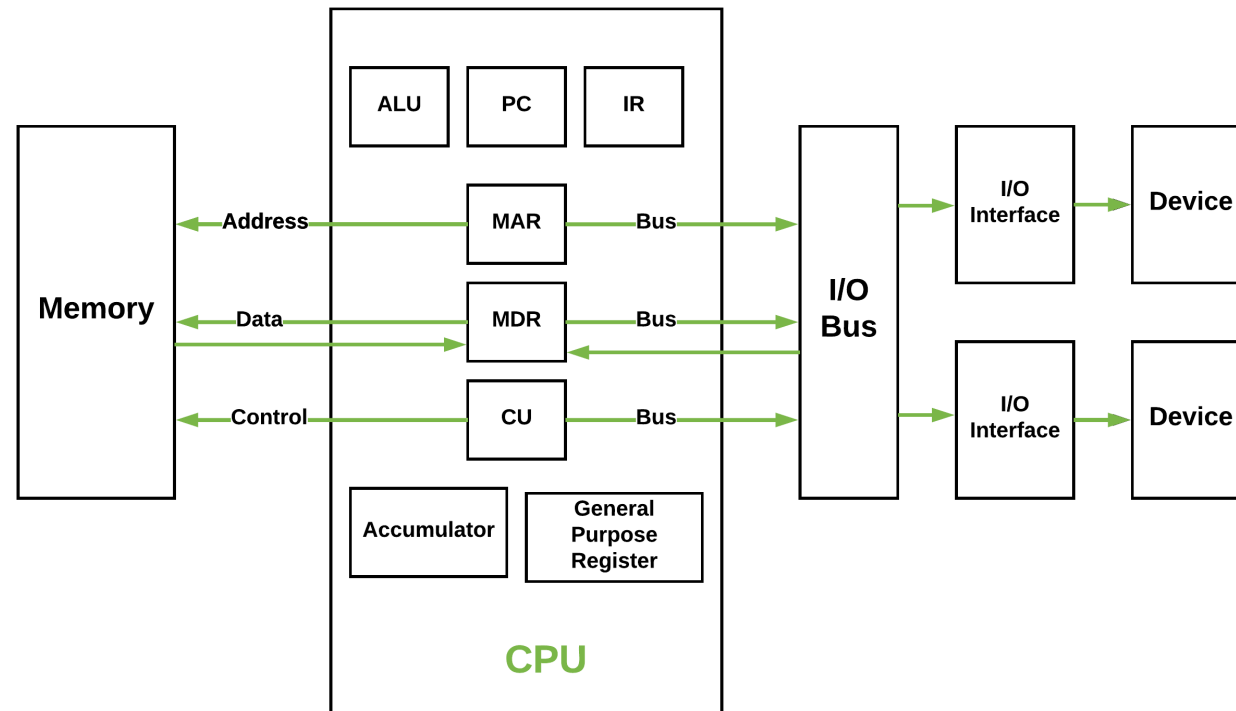


# Von Neumann Computer Vs. Restaurant



# Details in Von-Neumann Model

---



# Program Representation and Execution

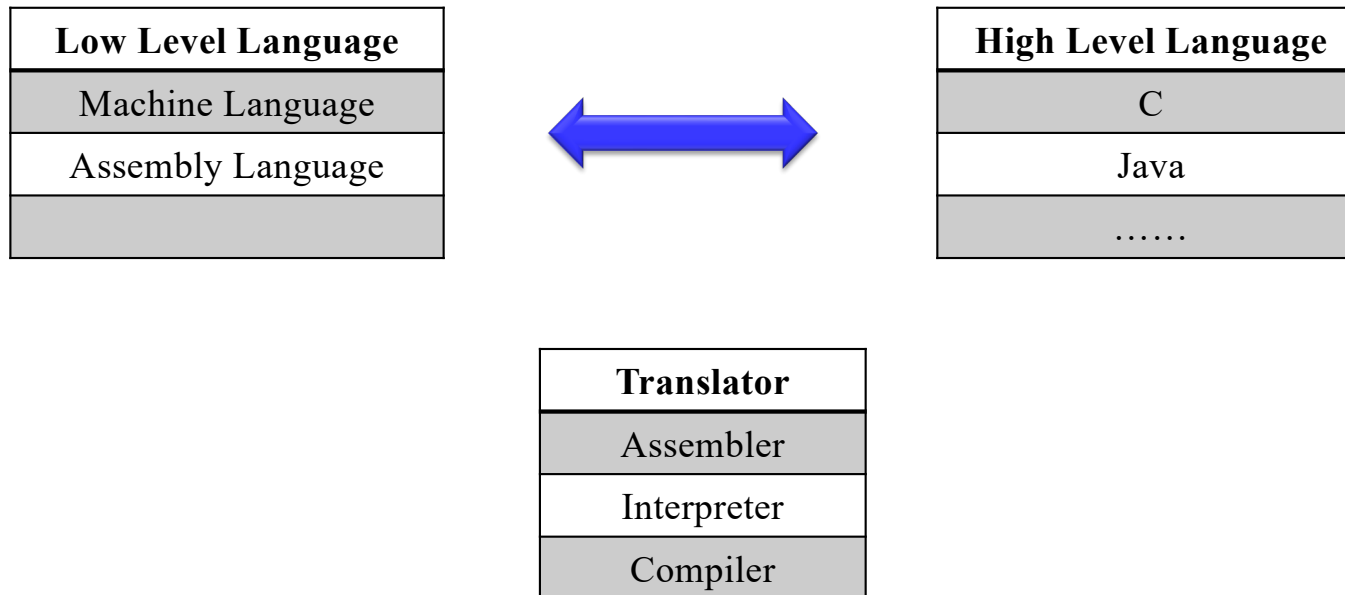
---

- **Program Representation**
- **Program Execution**

# Program Representation

---

- **Instructions**
  - 0/1 binaries
- **Program Language**



# Program Execution

---

- **Execute Instructions Automatically**
  - **Instruction Fetch**
  - **Instruction Decode**
  - **Execute**
  - **Memory Access**
  - **Write Back**

# Overview

---

- Introduction to computer systems
- **Binary number representation**
- Arithmetic operations
- Representation of numeric data
- Representation of non-numeric data
- Data width and storage

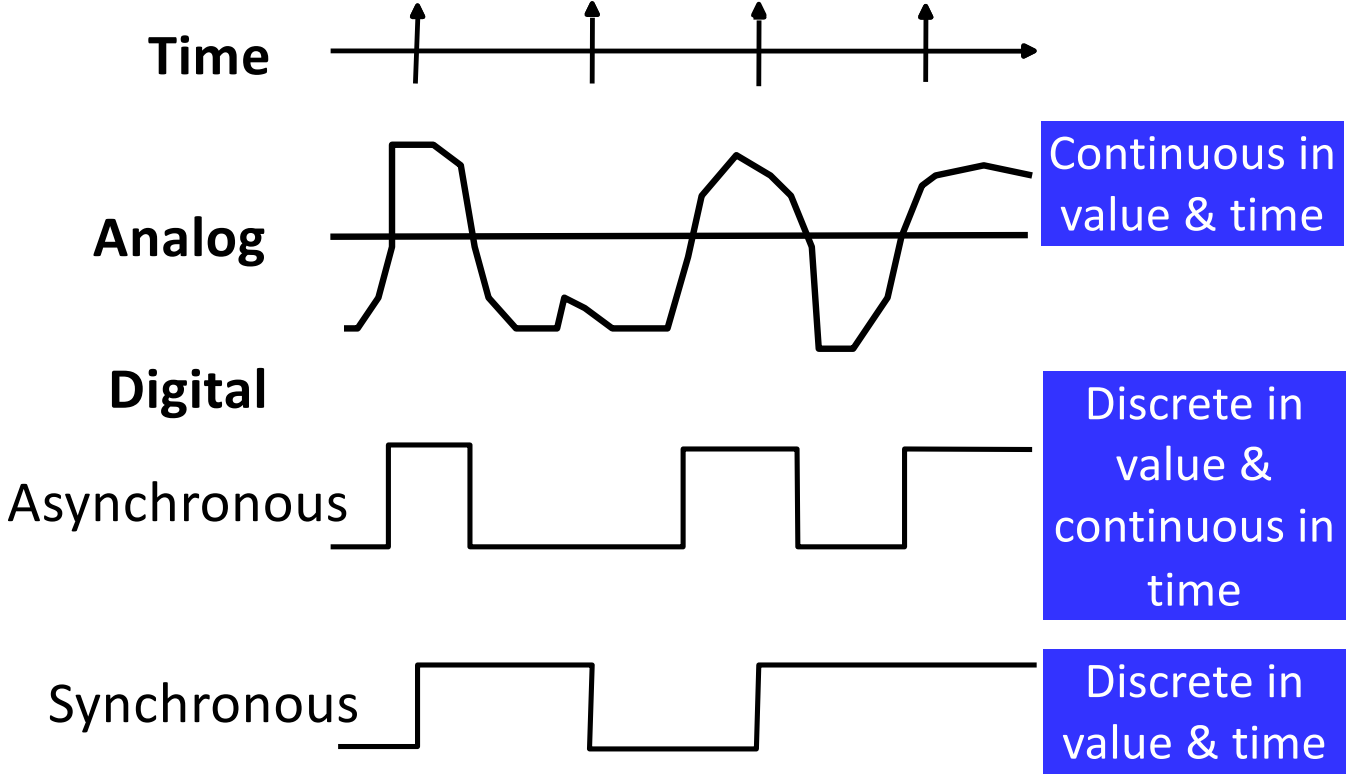
# INFORMATION REPRESENTATION - Signals

---

- Information variables represented by **physical quantities**.
- For digital systems, the variables take on **discrete** values.
- Two level, or binary values are the most prevalent values in digital systems.
- Binary values are represented abstractly by:
  - digits 0 and 1
  - words (symbols) False (F) and True (T)
  - words (symbols) Low (L) and High (H)
  - and words On and Off.
- Binary values are represented by values or **ranges of values** of physical quantities


# Signal Examples Over Time

---



# Signal Example – Physical Quantity: Voltage

---

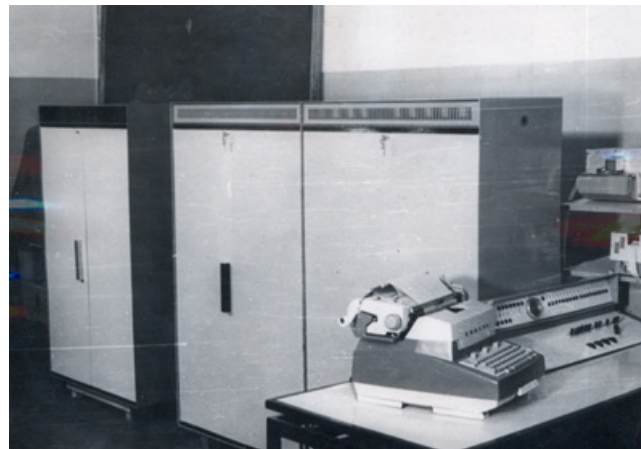
  
**Threshold  
Region**

# Why Binary?

---

- **Setun (Сету́нь)**

- **A balanced ternary computer developed in 1958 at Moscow State University**
- **The only modern ternary computer, using three-valued ternary logic**



Setun-70 in 1970

Is binary the most efficient one in digital systems?

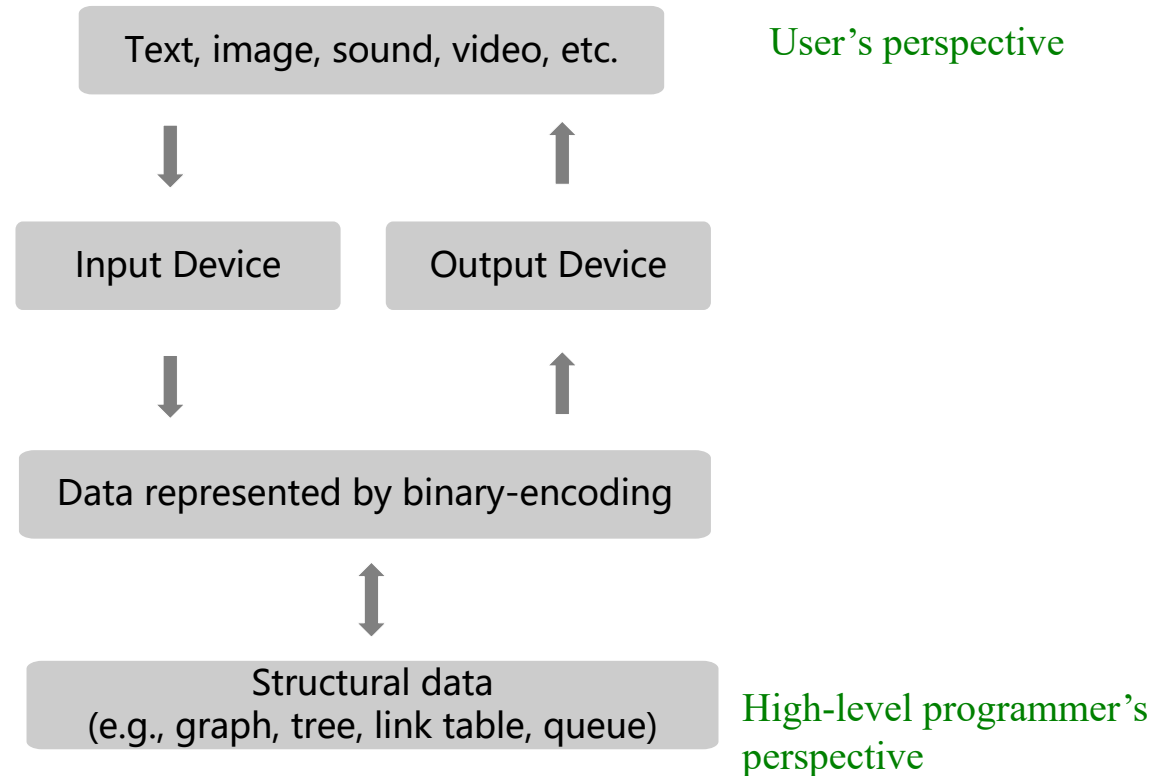
# External information and internal data

---

- **Continuous vs. discrete**
- **Advanced types vs. primitive types**

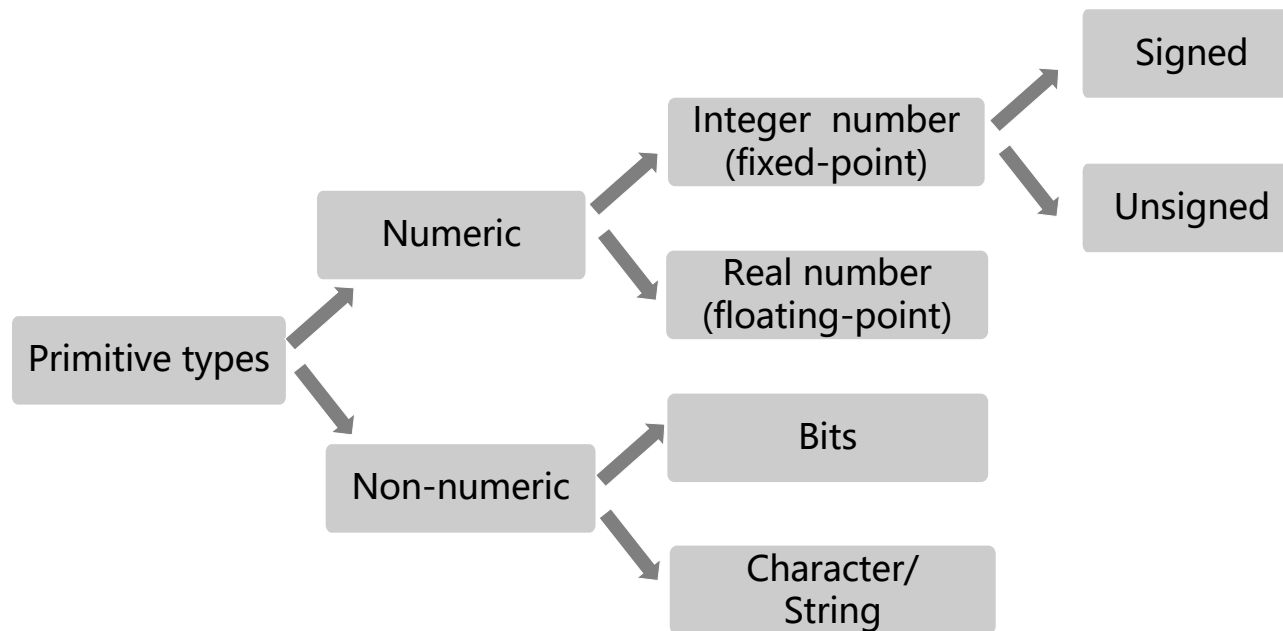
# External Information

---



# Internal Data

---



Low-level programmer's/  
hardware system designer's  
perspective

# Carry Counting System

---

- Positive radix, positional number systems
- A number with *radix*  $r$  is represented by a string of digits:

$A_{n-1}A_{n-2} \dots A_1A_0 \cdot A_{-1}A_{-2} \dots A_{-m+1}A_{-m}$   
in which  $0 \leq A_i < r$  and  $\cdot$  is the *radix point*.

- The string of digits represents the power series:

$$\begin{aligned} (\text{Number})_r &= \left( \sum_{i=0}^{n-1} A_i \cdot r^i \right) + \left( \sum_{j=-m}^{-1} A_j \cdot r^j \right) \\ &\quad \text{(Integer Portion)} + \text{(Fraction Portion)} \end{aligned}$$

# About the Decimal Number

---

**123.456**

$$1*10^2+2*10^1+3*10^0+4*10^{-1}+5*10^{-2}+6*10^{-3}$$

# What About the Binary Number?

---

$(100101.01)_2$

$$1*2^5+0*2^4+0*2^3+1*2^2+0*2^1+1*2^0+0*2^{-1}+1*2^{-2}$$

## Different Radixes

---

<b>Name</b>	<b>Radix</b>	<b>Digits</b>
<b>Binary</b>	<b>2</b>	<b>0,1</b>
<b>Octal</b>	<b>8</b>	<b>0,1,2,3,4,5,6,7</b>
<b>Decimal</b>	<b>10</b>	<b>0,1,2,3,4,5,6,7,8,9</b>
<b>Hexadecimal</b>	<b>16</b>	<b>0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F</b>

# Different Radixes

---

<b>Decimal (Base 10)</b>	<b>Binary (Base 2)</b>	<b>Octal (Base 8)</b>	<b>Hexadecimal (Base 16)</b>
<b>00</b>	<b>00000</b>	<b>00</b>	<b>00</b>
<b>01</b>	<b>00001</b>	<b>01</b>	<b>01</b>
<b>02</b>	<b>00010</b>	<b>02</b>	<b>02</b>
<b>03</b>	<b>00011</b>	<b>03</b>	<b>03</b>
<b>04</b>	<b>00100</b>	<b>04</b>	<b>04</b>
<b>05</b>	<b>00101</b>	<b>05</b>	<b>05</b>
<b>06</b>	<b>00110</b>	<b>06</b>	<b>06</b>
<b>07</b>	<b>00111</b>	<b>07</b>	<b>07</b>
<b>08</b>	<b>01000</b>	<b>10</b>	<b>08</b>
<b>09</b>	<b>01001</b>	<b>11</b>	<b>09</b>
<b>10</b>	<b>01010</b>	<b>12</b>	<b>0A</b>
<b>11</b>	<b>01011</b>	<b>13</b>	<b>0B</b>
<b>12</b>	<b>01100</b>	<b>14</b>	<b>0C</b>
<b>13</b>	<b>01101</b>	<b>15</b>	<b>0D</b>
<b>14</b>	<b>01110</b>	<b>16</b>	<b>0E</b>
<b>15</b>	<b>01111</b>	<b>17</b>	<b>0F</b>
<b>16</b>	<b>10000</b>	<b>20</b>	<b>10</b>

## Converting R-base to Decimal

---

- To convert to decimal, use decimal arithmetic to form  $\Sigma$  (digit  $\times$  respective power of R).
- Convert  $(100101.01)_2$  to  $N_{10}$ :
  - $1*2^5+0*2^4+0*2^3+1*2^2+0*2^1+1*2^0+0*2^{-1}+1*2^{-2}=(37.25)_{10}$
- Convert  $3A.CH$  to  $N_{10}$ :
  - $3*16^1+10*16^0+12*16^{-1}=(58.75)_{10}$

# Converting Decimal to R-base

---

- **To convert from one base to another:**
  - **Convert the integer part**
  - **Convert the fraction part**
  - **Join the two results together**

## Convert the Integral Part

---


- Repeatedly divide the number by the new radix and save the remainders. The digits for the new radix are the remainders in *reverse order* of their computation.
  - If the new radix is  $> 10$ , then convert all remainders  $> 10$  to digits A, B, ...

# Example: Convert $135_{10}$ To Base 2

---

$$(135)_{10} = (1000\ 0111)_2$$

2		1	3	5	
2		6	7	.....	1
2		3	3	.....	1
2		1	6	.....	1
2		8	.....	0	
2		4	.....	0	
2		2	.....	0	
2		1	.....	0	
2		0	.....	1	



## Convert the Fractional Part

---

- Repeatedly multiply the fraction by the new radix and save the integer digits that result. The digits for the new radix are the integer digits in *order* of their computation.
  - If the new radix is  $> 10$ , then convert all remainders  $> 10$  to digits A, B, ...

## Example: Convert $0.6875_{10}$ To Base 2

---

$$(0.6875)_{10} = (0.1011)_2$$

$$\begin{array}{l} \underline{2 \times 0.6875 \dots\dots\dots} = 1.375 \\ \underline{2 \times 0.375 \dots\dots\dots} = 0.75 \\ \underline{2 \times 0.75 \dots\dots\dots} = 1.5 \\ 2 \times 0.5 \dots\dots\dots = 1 \end{array}$$



## Convert $135.6875_{10}$ To Base 2

---

- Convert 135 to Base 2

$$(135)_{10} = (10000111)_2$$

- Convert 0.6875 to Base 2:

$$(0.6875)_{10} = (0.1011)_2$$

- **Join the results together**

$$(135.6875)_{10} = (10000111.1011)_2$$

## Additional Issue - Fractional Part

---

- Note that in this conversion, the fractional part can become 0 as a result of the repeated multiplications.
- In general, it may take many bits to get this to happen or it may never happen.
- Example Problem: Convert  $0.65_{10}$  to  $N_2$ 
  - $0.65 = 0.1010011001001 \dots$
  - The fractional part begins repeating every 4 steps yielding repeating 1001 forever!
- Solution: Specify number of bits to right of radix point and round or truncate to this number.

# Why Do Repeated Division and Multiplication Work?

---

- Divide the integer portion of the power series by radix  $r$ . The remainder of this division is  $A_0$ , represented by the term  $A_0/r$ .
- Discard the remainder and repeat, obtaining remainders  $A_1, \dots$
- Multiply the fractional portion of the power series by radix  $r$ . The integer part of the product is  $A_{-1}$ .
- Discard the integer part and repeat, obtaining integer parts  $A_{-2}, \dots$
- This demonstrates the algorithm for any radix  $r > 1$ .

# Octal (Hexadecimal) to Binary and Back

---

- **Octal (Hexadecimal) to Binary:**
  - **Restate the octal (hexadecimal) as three (four) binary digits starting at the radix point and going both ways.**
- **Binary to Octal (Hexadecimal):**
  - **Group the binary digits into three (four) bit groups starting at the radix point and going both ways, padding with zeros as needed in the fractional part.**
  - **Convert each group of three bits to an octal (hexadecimal) digit.**

## Hexadecimal to Binary and Back

---

(0)H = 0000	(1)H = 0001
(2)H = 0010	(3)H = 0011
(4)H = 0100	(5)H = 0101
(6)H = 0110	(7)H = 0111
(8)H = 1000	(9)H = 1001
(A)H = 1010	(B)H = 1011
(C)H = 1100	(D)H = 1101
(E)H = 1110	(F)H = 1111

## Some Examples

---

$$(67.731)_8 = (110\ 111\ .111\ 011\ 001)_2$$

$$(312.64)_8 = (011\ 001\ 010\ .\ 110\ 1)_2$$

$$(11\ 111\ 101\ .\ 010\ 011\ 11)_2 = (375.236)_8$$

$$(10\ 110.11)_2 = (26.6)_8$$

$$(2B.5E)_{16} = (0010\ 1011.0101\ 1110)_2$$

$$(21A.5)_{16} = (0010\ 0001\ 1010\ .\ 0101)_2$$

$$(1001101.01101)_2 = (0100\ 1101.\ 0110\ 1000)_2 = (4D.68)_{16}$$

$$(110\ 0101.101)_2 = (65.A)_{16}$$

# Octal to Hexadecimal via Binary

---

- Convert octal to binary.
- Use groups of four bits and convert as above to hexadecimal digits.
- Example: Octal to Binary to Hexadecimal

6    3    5 . 1    7    7    8

➔ 110 | 011 | 101 . 001 | 111 | 111<sub>2</sub>

➔ 1 | 1001 | 1101 . 0011 | 1111 | 1(000)<sub>2</sub>

➔ 1    9    D . 3    F    8<sub>16</sub>

**Why do these conversions work?**

## Simple BASE CONVERSION(From Decimal to Binary) - Positive Powers of 2

---

- Useful for Base Conversion

Exponent	Value
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Exponent	Value
11	2,048
12	4,096
13	8,192
14	16,384
15	32,768
16	65,536
17	131,072
18	262,144
19	524,288
20	1,048,576
21	2,097,152

## Special Powers of 2

---

- $2^{10}$  (1024) is Kilo, denoted "K"
- $2^{20}$  (1,048,576) is Mega, denoted "M"
- $2^{30}$  (1,073, 741,824) is Giga, denoted "G"
- $2^{40}$  (1,099,511,627,776) is Tera, denoted "T"

## 2<sup>X</sup> vs. 10<sup>Y</sup> Bytes Ambiguity

---

Decimal term	Abbreviation	Value	Binary term	Abbreviation	Value	% Larger
kilobyte	KB	10 <sup>3</sup>	kibibyte	KiB	2 <sup>10</sup>	2%
megabyte	MB	10 <sup>6</sup>	mebibyte	MiB	2 <sup>20</sup>	5%
gigabyte	GB	10 <sup>9</sup>	gibibyte	GiB	2 <sup>30</sup>	7%
terabyte	TB	10 <sup>12</sup>	tebibyte	TiB	2 <sup>40</sup>	10%
petabyte	PB	10 <sup>15</sup>	pebibyte	PiB	2 <sup>50</sup>	13%
exabyte	EB	10 <sup>18</sup>	exbibyte	EiB	2 <sup>60</sup>	15%
zettabyte	ZB	10 <sup>21</sup>	zebibyte	ZiB	2 <sup>70</sup>	18%
yottabyte	YB	10 <sup>24</sup>	yobibyte	YiB	2 <sup>80</sup>	21%

# Overview

---

- Introduction to computer systems
- Binary number representation
- **Arithmetic operations**
- Representation of numeric data
- Representation of non-numeric data
- Data width and storage

## ARITHMETIC OPERATIONS - Binary Arithmetic

---

- **Single bit addition with carry**
- **Multiple bit addition**
- **Single bit subtraction with borrow**
- **Multiple bit subtraction**
- **Multiplication**
- **Division**

# Single Bit Binary Addition with Carry

---

Given two binary digits (X,Y), a carry in (Z) we get the following sum (S) and carry (C):

Carry in (Z) of 0:

Z	0	0	0	0
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
C S	0 0	0 1	0 1	1 0

Carry in (Z) of 1:

Z	1	1	1	1
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
C S	0 1	1 0	1 0	1 1

## Multiple Bit Binary Addition

---

- Extending this to two multiple bit examples:

<b>Carries</b>	<b><u>0</u></b>	<b><u>0</u></b>
<b>Augend</b>	<b>01100</b>	<b>10110</b>
<b>Addend</b>	<b><u>+10001</u></b>	<b><u>+10111</u></b>
<b>Sum</b>	<b>11100</b>	<b>101101</b>

- Note: The 0 is the default Carry-In to the least significant bit.

## Single Bit Binary Subtraction with Borrow

---

- Given two binary digits (X,Y), a borrow in (Z) we get the following difference (S) and borrow (B):

- Borrow in (Z) of 0:
 

Z	0	0	0	0
---	---	---	---	---

X	0	0	1	1
---	---	---	---	---

<u>-Y</u>	<u>-0</u>	<u>-1</u>	<u>-0</u>	<u>-1</u>
-----------	-----------	-----------	-----------	-----------

B S	0 0	1 1	0 1	0 0
-----	-----	-----	-----	-----

- Borrow in (Z) of 1:
 

Z	1	1	1	1
---	---	---	---	---

X	0	0	1	1
---	---	---	---	---

<u>-Y</u>	<u>-0</u>	<u>-1</u>	<u>-0</u>	<u>-1</u>
-----------	-----------	-----------	-----------	-----------

B S	1 1	1 0	0 0	1 1
-----	-----	-----	-----	-----

## Multiple Bit Binary Subtraction

---

- Extending this to two multiple bit examples:

<b>Borrows</b>	<b><u>0</u></b>	<b><u>0</u></b>
<b>Minuend</b>	<b>10110</b>	<b>10110</b>
<b>Subtrahend</b>	<b>- <u>10010</u></b>	<b>- <u>10011</u></b>
<b>Difference</b>	<b>00100</b>	<b>00011</b>

- **Notes:** The 0 is a Borrow-In to the least significant bit. If the Subtrahend > the Minuend, interchange and append a – to the result.

# Binary Multiplication

---

The binary multiplication table is simple:

$$0 * 0 = 0 \quad | \quad 1 * 0 = 0 \quad | \quad 0 * 1 = 0 \quad | \quad 1 * 1 = 1$$

Extending multiplication to multiple digits:

<b>Multiplicand</b>	<b>1011</b>
<b>Multiplier</b>	<b><u>x 101</u></b>
<b>Partial Products</b>	<b>1011</b>
	<b>0000 -</b>
	<b><u>1011 - -</u></b>
<b>Product</b>	<b>110111</b>

## Binary Division

---

The binary multiplication table is simple:

$$0 / 0 \quad | \quad 1 / 0 \quad \text{meaningless}$$

$$0 / 1 = 0 \quad | \quad 1 / 1 = 1$$

Extending division to multiple digits:

<b>Dividend</b>	11001
<b>Divisor</b>	<u>   / 101</u>
	101) 11001
	101
	<u>   </u>
	101
	<u>   </u>
	101
	<u>   </u>
<b>Quotient</b>	101

## A Final Conversion Note

---

- You can use arithmetic in other bases if you are careful:
- Example: Convert  $101110_2$  to Base 10 using binary arithmetic:

$$\text{Step 1 } 101110 / 1010 = 100 \text{ r } 0110$$

$$\text{Step 2 } 100 / 1010 = 0 \text{ r } 0100$$

Converted Digits are  $0100_2$  |  $0110_2$

or 4 6<sub>10</sub>